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Uvod u vjerojatnost i statistiku

Na ovim slajdovima je samo mali dio aktualne prezentacije

i	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	506	-20.6	424.36
2	517	-9.6	92.16
3	545	18.4	338.56
4	545	18.4	338.56
5	520	-6.6	43.56
Σ	2633	0	1237.2

$$\Sigma_{i=1}^{5} x_i = 506 + 517 + 545 + 545 + 520$$

Srednja vrijednost

$$\bar{x} = \frac{1}{n} \sum_i x_i = \frac{1}{5} \times 2633 = 526.6$$

i	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	506	-20.6	424.36
2	517	-9.6	92.16
3	545	18.4	338.56
4	545	18.4	338.56
5	520	-6.6	43.56
Σ	2633	0	1237.2

Varijanca = srednje kvadratno odstupanje

$$Var(x) = \frac{1}{5}\Sigma_{i=1}^{5}(x_i - \bar{x})^2 = \frac{1237.2}{5} = 247.44$$

Standardna devijacija $\sigma_x = \sqrt{Var(x)} = 15.73$

Rezultat mjerenja: $x = \bar{x} \pm \sigma_x$

$$\boxed{x = 526.6 \pm 15.7}$$

i	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	231	9	81
2	213	-9	81
3	209	-13	169
4	251	29	841
5	206	-16	256
Σ	1110	0	1428

Srednja vrijednost

$$\bar{x} = \frac{1}{5}\Sigma_{i=1}^{5} x_i = 1110/5 = 222$$

Standardna devijacija

$\sigma_x = \sqrt{Var(x)} = \sqrt{1428/5} = 16.8997$

Rezultat mjerenja:

$x = \bar{x} \pm \sigma_x$

$\boxed{x = 222 \pm 17}$

i	x_i	f_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1				
2				
3				
4				
5				
Σ				

Srednja vrijednost

$$\bar{x} = \frac{\Sigma_{i=1}^{5} x_i f_i}{\Sigma_{i=1}^{5} f_i} =$$

Standardna devijacija
$\sigma_x = \sqrt{Var(x)} =$

Rezultat mjerenja:
$x = \bar{x} \pm \sigma_x$

$\boxed{x =}$

i	f_i	x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	3	19	-2.846	8.099716
2	2	20	-1.846	3.407716
3	4	21	-0.846	0.715716
4	1	25	3.154	9.947716
5	3	26	4.154	17.255716
Σ	13	284	0.002	95.692308

Srednja vrijednost

$$\Sigma_{i=1}^{5} x_i \cdot f_i = 284$$

$$n = \sum_i f_i = 13$$

$$\bar{x} = \frac{1}{n}\Sigma_{i=1}^{5} x_i = \frac{284}{13} = 21.846$$

$$Var(x) = 95.692308/13 = 7.3609467$$

Standardna devijacija
$\sigma_x = \sqrt{Var(x)} = 2.713$

Rezultat mjerenja:
$x = \bar{x} \pm \sigma_x$

$$\boxed{x = 21.846 \pm 2.713}$$

x_i	$\triangle x_i$	y_i	$\triangle y_i$	$(\triangle x_i)^2$	$(\triangle y_i)^2$	$(\triangle x_i)(\triangle y_i$
142.71	-2,13	220.58	4,13	4,5369	17,0569	-8,7969
143.72	-1,12	220.01	3,56	1,2544	12,6736	-3,9872
141.61	-3,23	219.57	3,12	10,4329	9,7344	-10,0776
147.29	2,45	212.47	-3,98	6,0025	15,8404	-9,751
148.93	4,09	206.58	-9,87	16,7281	97,4169	-40,3683
144.78	-0,06	219.49	3,04	0,0036	9,2416	-0,1824
869.04	0	1298.70	0	38.9584	161.9638	-73.1634
144.84		216.45		6.4931	26.9940	-12.1939

kovarijanca para slučajnih veličina, (x, y),

je izraz

$$Cov(x, y) = \frac{1}{n} \sum_i (x_i - \overline{x}) \cdot (y_i - \overline{y})$$

$$= \frac{(x_1 - \overline{x}) \cdot (y_1 - \overline{y}) + (x_2 - \overline{x}) \cdot (y_2 - \overline{y}) + \ldots + (x_n - \overline{x}) \cdot (y_n - \overline{y})}{n}$$

gdje je n broj eksperimenata i u i-tom eksperimentu smo izmjerili (x_i, y_i). Između ostalog, $Var(x) = Cov(x, x)$.

pravac regresije

$$y - \overline{y} = \frac{Cov(x, y)}{Var(x)}(x - \overline{x})$$

$$y - 216.45 = \frac{-12.1939}{6.4931}(x - 144.84)$$

$$y = -1.877978x + 488.43$$

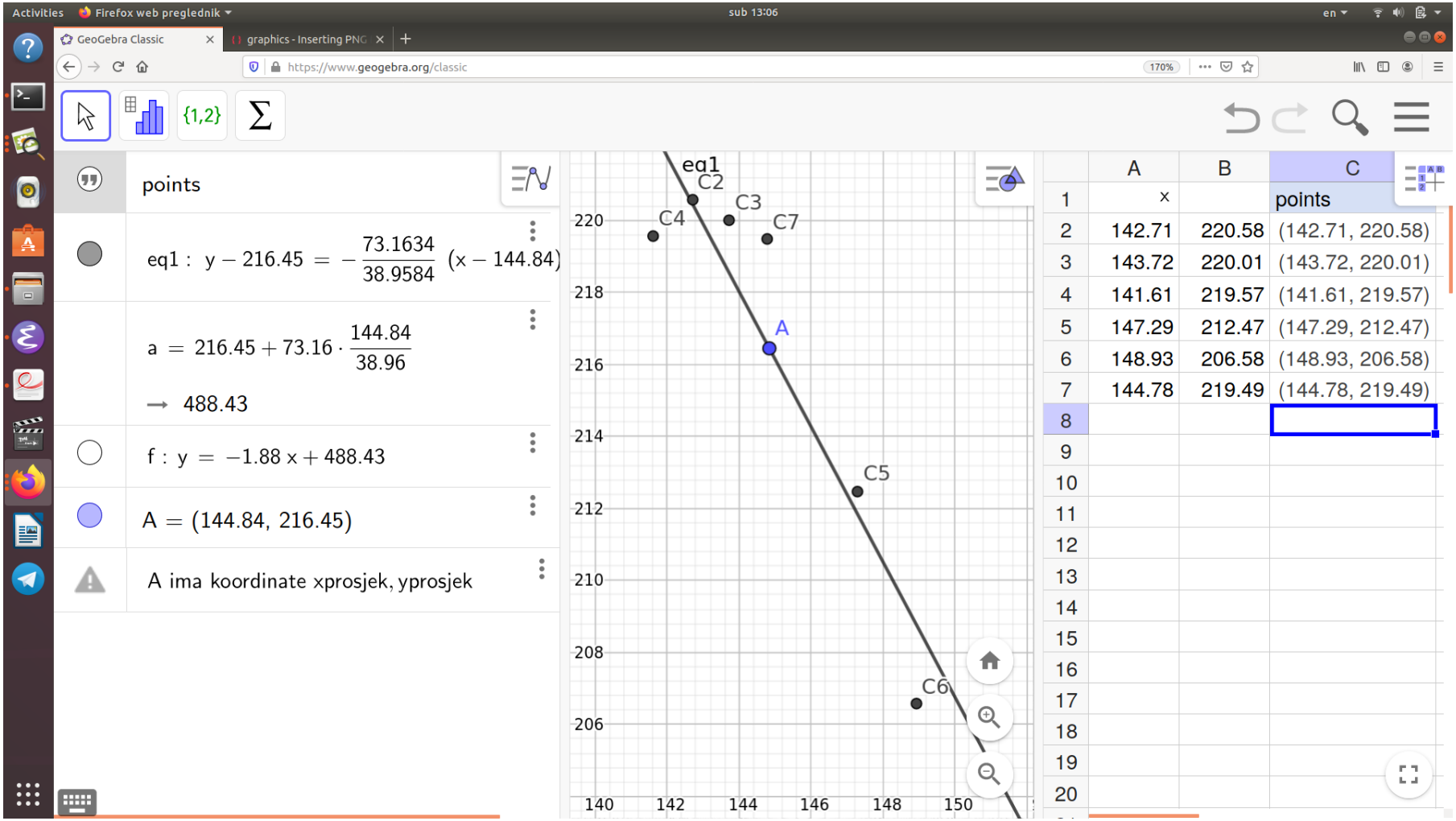
korelacija x i y

$$Kor(x,y) = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_j (x_j - \bar{x})^2}\sqrt{\sum_k (y_k - \bar{y})^2}}$$

Ako gore i dolje podijelimo s n, to je isto što i

$$\frac{Cov(x,y)}{\sqrt{Var(x)Var(y)}} = \frac{Cov(x,y)}{\sigma_x \cdot \sigma_y}$$

$$Kor(x,y) = \frac{-12.1939}{13.239} = -0.921$$


Activities
Firefox web preglednik
sub 13:06
en
GeoGebra Classic
graphics - Inserting PNG
https://www.geogebra.org/classic
170%
points
eq1 : y − 216.45 = − 73.1634/38.9584 (x − 144.84)
a = 216.45 + 73.16 · 144.84/38.96
→ 488.43
f : y = −1.88 x + 488.43
A = (144.84, 216.45)
A ima koordinate xprosjek, yprosjek
eq1
C2
C3
C4
C7
A
C5
C6
220
218
216
214
212
210
208
206
140
142
144
146
148
150
A
B
C
points
1 x
2 142.71 220.58 (142.71, 220.58)
3 143.72 220.01 (143.72, 220.01)
4 141.61 219.57 (141.61, 219.57)
5 147.29 212.47 (147.29, 212.47)
6 148.93 206.58 (148.93, 206.58)
7 144.78 219.49 (144.78, 219.49)
8
9
10
11
12
13
14
15
16
17
18
19
20

x_i	y_i	$\triangle x_i$	$\triangle y_i$	$(\triangle x_i)^2$	$(\triangle y_i)^2$	$(\triangle x_i)(\triangle y_i)$
6.84	7.13	-3.27	4.01	10.6929	16.0801	-13.1127
9.10	6.68	-1.01	3.56	1.0201	12.6736	-3.5956
9.33	4.28	-0.78	1.16	0.6084	1.3456	-0.9048
12.56	-0.86	2.45	-3.98	6.0025	15.8404	-9.7510
13.23	-4.30	3.12	-7.42	9.7344	55.0564	-23.1504
9.60	5.79	-0.51	2.67	0.2601	7.1289	- 1.3617
60.66	18.72	0	0	28.3184	108.1250	-51.8762
10.11	3.12			4.7197	18.0208	-8.6460

$$\sigma_x = \sqrt{Var(x)} = \sqrt{4.7197} = 2.172487$$

$$\sigma_y = \sqrt{Var(y)} = \sqrt{18.0208} = 4.24509$$

$$x = \bar{x} \pm \sigma_x = 10.11 \pm 2.17$$

$$y = \bar{y} \pm \sigma_y = 3.12 \pm 4.25$$

Koeficijent korelacije je dakle

$$Kor(x,y) = \frac{-8.6460}{2.172487 \cdot 4.24509} = -0.9362,$$

a koeficijent regresije je

$$k = \frac{-51.8762}{28.3184} = \frac{-8.6460}{4.7197} = -1.83$$

pa je jednadžba pravca regresije

$$x - 3.12 = -1.83(x - 10.11)$$

ili $y = -1.83x - 3.12 + 1.83 \cdot 10.11 = -1.83x + 15.40$