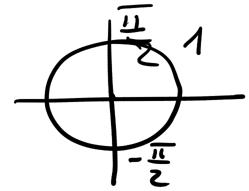
mat3 2.2.2023. IME i PREZIME:



Apsolutno zabranjeni kalkulatori i mobiteli

1. Nadji domenu realne funkcije $f(x) = \tan(\frac{2}{x})$.

$\frac{2}{x} \leftarrow \quad x \neq 0 \qquad \tan\frac{2}{x} = \frac{\sin\frac{2}{x}}{\cos\frac{2}{x}} \leftarrow 0$ kad je $\frac{2}{x} \neq \frac{\pi}{2} + k\pi$, $k \in \mathbb{Z}$

$\frac{2}{\frac{\pi}{2} + k\pi} \neq x$

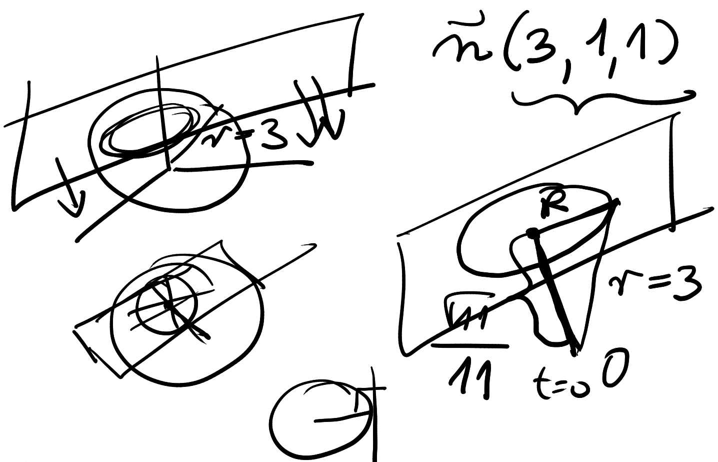
$D_f = \mathbb{R} \setminus \left(\{0\} \cup \left\{ \frac{2}{\frac{\pi}{2} + k\pi}, k \in \mathbb{Z} \right\} \right)$

2. Nadji površinu kruga koji se dobije kao presjek ravnine $3x+y+z+1=0$ M s kuglom radijusa $r = 3$ sa središtem u ishodištu.

$\vec{n} \cdot (\vec{r} - \vec{r}_A) = 0$

$n_x x + n_y y + n_z z + d = 0$

$\vec{n}(3,1,1)$



$A + t\vec{n} \in M$

$(0,0,0) + t(3,1,1) \in M$

$3 \cdot 3t + 1 \cdot 1t + 1 \cdot 1t + 1 = 0$

$3x + y + z + 1 = 0$

$11t = -1$

$t = -\frac{1}{11} \qquad d\left(-\frac{1}{11}(3,1,1), 0\right) = \frac{1}{11}\sqrt{\underbrace{3^2+1^2+1^2}_{11}} = \frac{\sqrt{11}}{11}$

$R = \sqrt{r^2 - \frac{1}{11}} = \sqrt{9 - \frac{1}{11}} = \sqrt{\frac{98}{11}}$

3. Gaussovom metodom eliminacije riješi sustav jednadžbi $-1, 2, \frac{1}{2}$

$$
\begin{aligned}
2x + y - z &= -1/2 \\
-x + y + z &= 7/2 \\
x - y - 2z &= -4
\end{aligned}
$$

$$
\left.\begin{aligned}
x - y - 2z &= -4 \quad = -8/2 \\
-x + y + z &= 7/2 \\
2x + y - z &= -1/2
\end{aligned}\right\}
\begin{aligned}
&\\
&+\mathrm{I} \\
&-2\mathrm{I}
\end{aligned}
$$

$$y - 2(-y)$$

$$
\begin{aligned}
x - y - 2z &= -4 \\
-z &= -1/2 \\
3y + 3z &= \frac{15}{2} \quad /:3
\end{aligned}
$$

$$-1/2 - 2(-4) = -\frac{1}{2} + 8 = \frac{-1+16}{2}$$

$$
\left\{\begin{aligned}
x - y - 2z &= -4 \\
y + z &= \frac{5}{2} \\
z &= \frac{1}{2}
\end{aligned}\right.
\begin{aligned}
&+2\mathrm{III} \\
&-\mathrm{III} \\
&
\end{aligned}
$$

$$
\begin{aligned}
x - y &= -3 \qquad x = -1 \\
y &= 2 \\
z &= \frac{1}{2}
\end{aligned}
$$

4. Koliko je

$3 \times 3 \times 3 = 27$

$\left(2^{\log_2 a}\right)^2 = a^2$

- $\cos(7\pi/3) =$ $\frac{1}{2}$
- $\log_{10}(100^{-5}) =$ $\log_{10}\left((10^2)^{-5}\right) = -10$
- $\frac{\log_2 3}{\log_4 27} =$ $\frac{\log_2 3}{\log_4 3^3} = \frac{\log_2 3}{\log_2 3^{3/2}} = \frac{\log_2 3}{\frac{3}{2}\log_2 3} = \frac{2}{3}$
- $\sqrt[3]{4^{15/2}} =$ $\sqrt[3]{4^{3\cdot 5/2}} = 4^{5/2} = 2^5 = 2\cdot 2\cdot 2\cdot 2\cdot 2 = 32$
- $\mathrm{arctg}(-1) =$ $-\frac{\pi}{4}$
- Površina jednakostraničnog trokuta stranice $a = 2$ (na dvije decimale, tj. uzimamo da je $\pi = 3.14$)

$P = \frac{a\frac{a\sqrt{3}}{2}}{2} = \frac{a^2\sqrt{3}}{4}$

$\frac{a\sqrt{3}}{2} = \sqrt{a^2 - \left(\frac{a}{2}\right)^2}$

$2\log_2 a = a^2$

$4^{\log_2 a} = a^2$

$\log a^b = b \log a$

$\sqrt{3} = 1.74$

$\frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$

60°

$\sqrt{3{,}00\,00} = 1{,}74$

$2\,00 : 27$

$16\,00 : 344$

-
$$\begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 \\ -3 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 3 & -3 \end{pmatrix} =$$

$$\begin{pmatrix} -5 & -11 \\ 10 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 3 & -3 \end{pmatrix} = \begin{pmatrix} -6 & -12 \\ 7 & 5 \end{pmatrix}$$

5. Nadji skup $S \subseteq \mathbf{R}$ svih rješenja nejednadžbe $(x+3)(x-2)(x+7) > 0$.

+	+	+
+	−	−
−	−	+
−	+	−

1. $x > -3$, $x > 2$, $x > -7$ $\quad \langle 2, \infty\rangle$
2. $x > -3$, $x < 2$, $x < -7$ $\quad \emptyset$
3. $x < -3$, $x < 2$, $x > -7$ $\quad \langle -7, -3\rangle$

   $-7 \quad -3 \quad 2$
4. $x < -3$, $x > 2$, $x < -7$ $\quad \emptyset$

$S = \langle -7, -3\rangle \cup \langle 2, \infty\rangle$

6. Odredi sjecišta s koordinatnim osima (ako postoje), koordinate tjemena i skiciraj parabolu $y = 2x^2 + 3x - 2$.

$a > 0$

$\boxed{y = 0 \;\; \text{os } x}$

$2x^2 + 3x - 2 = 0$

$(x+a)^2 = x^2 + 2ax + a^2$

$2\left(x^2 + \frac{3}{2}x - 1\right) = 0$

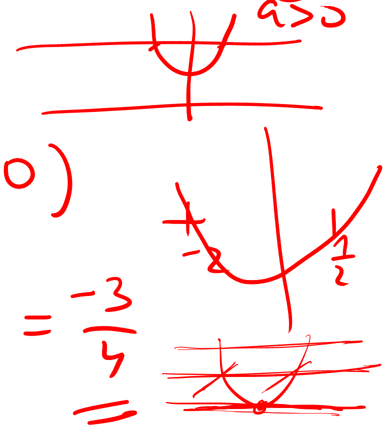
$x^2 + \frac{3}{2}x - 1 = 0$

$\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - 1 = 0$

$x + \frac{3}{4} = \frac{9}{16} + \frac{16}{16} = \frac{25}{16}$

$x = -\frac{3}{4} \pm \frac{5}{4}$ $\quad x_1 = \frac{1}{2}, \quad x_2 = -2$

$(-2, 0), \left(\frac{1}{2}, 0\right)$



$x_T = \frac{-2 + \frac{1}{2}}{2} = \frac{-3}{4}$

$\boxed{x_T + \frac{3}{4} = 0}$

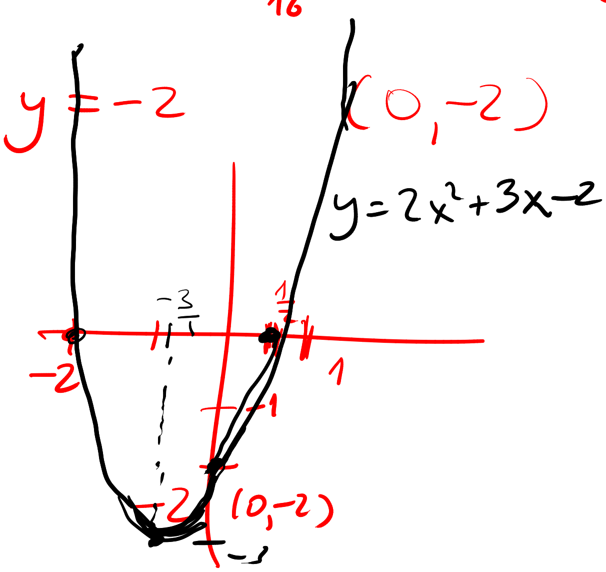
$y_T = 2x_T^2 + 3x_T - 2$

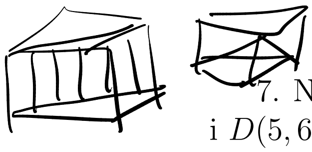
$= 2\left(\frac{+9}{16}\right) + 3\,\frac{-3}{4} - 2$

$= \frac{+18 - 36 - 2 \cdot 16}{16} = \frac{-18 - 32}{16} = -3$

os y … $x = 0$ $\quad y = -2$ $\quad (0, -2)$

$T\left(-\frac{3}{4}, -3\right)$





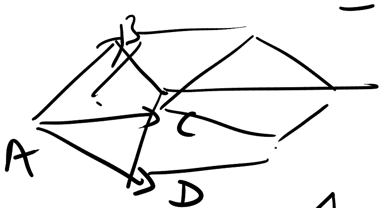
7. Nadji obujam piramide kojoj su vrhovi $A(2,0,-1)$, $B(3,2,3)$, $C(4,1,-1)$ i $D(5,6,3)$.

$$\pm\frac{1}{6}(\vec{AB}\times\vec{AC})\cdot\vec{AD}$$

$\vec{AB}(1,2,4)$

$\vec{AC}(2,1,0)$

$\vec{AD}(3,6,4)$



$$\frac{1}{6}\begin{vmatrix}1&2&4\\2&1&0\\3&6&4\end{vmatrix} = \frac{1}{6}\left(4\begin{vmatrix}2&1\\3&6\end{vmatrix} + 4\begin{vmatrix}1&2\\2&1\end{vmatrix}\right)$$

$$Vol = \frac{1}{6}(4\cdot 9 + 4(-3)) = 4$$

8. Nadji sve druge korijene kompleksnog broja $3+i$ (realni i imaginarni dio oba rješenja) u terminima racionalnih brojeva i drgih korijena (realnih) racionalnih brojeva.

$$\sqrt{3+i} = u+vi \quad /^2$$

$$3+i = u^2+2uvi-v^2$$

$$u^2-v^2=3 \qquad u^2-v^2=3$$

$$2uvi = i \qquad 2uv = 1$$

$$v = \frac{1}{2u}$$

$$u^2 - \frac{1}{4u^2} = 3 \quad /\cdot 4u^2 \neq 0$$

$$u^2 > 0$$

$$u^2 - \frac{1}{4u^2} = 3 \quad /\cdot 4u^2$$

$$4u^4 - 3\cdot 4\cdot u^2 - 1 = 0$$

$$4u^4 - 12u^2 - 1 = 0$$

$$u^2 = t > 0$$

$$4t^2 - 12t - 1 = 0$$

$$4\left(t^2 - 3t - \frac{1}{4}\right) = 0$$

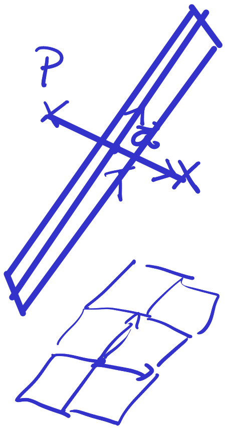
$$\left(t-\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - \frac{1}{4} = 0$$

$$t_{1,2} = \frac{12 \pm \sqrt{144+16}}{8}$$

$$t = +\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 + \frac{1}{4}}$$

$$= \frac{3}{2} + \sqrt{\frac{5}{2}}$$

$$u = \sqrt{t} \qquad v = \frac{1}{2u}$$



$\to \vec r(u,v) = T + u\overrightarrow{PQ} + v\vec a$

$\vec r(t) = A + t\vec a$

$Q \to \vec n = \overrightarrow{PQ}$ $\quad ax+by+cz+d=0$

9. Nadji jednadžbu ravnine koja je geometrijsko mjesto svih točaka koje su jednako udaljene od $P(2,0,3)$ i $Q(1,5,6)$, drugim riječima ravnine koja je okomita na dužinu $\overline{PQ}$ i prolazi kroz njeno polovište.

$T\left(\frac{2+1}{2}, \frac{0+5}{2}, \frac{3+6}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}, \frac{9}{2}\right)$

$\overrightarrow{PQ}(-1,5,3)$

$-x+5y+3z+d=0$

$-\frac{3}{2} + 5\frac{5}{2} + 3\frac{9}{2} + d = 0$

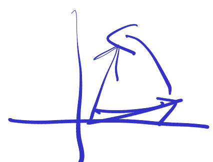
$d = -\frac{49}{2}$

$-3+25+27 = 49$

$\vec a = \overrightarrow{PQ} \times \vec i = \begin{vmatrix} -1 & 5 & 3 \\ 1 & 0 & 0 \\ \vec i & \vec j & \vec k \end{vmatrix} = -1\begin{vmatrix} 5 & 3 \\ \vec j & \vec k \end{vmatrix} = -5\vec k + 3\vec j$

10. Koji vektor $\vec v'$ dobijemo ako rotiramo radijusvektor $\vec v = 3\vec i + \vec j$ oko ishodišta u ravnini xy za 60 stupnjeva u smjeru kazaljke na satu ?

$\vec v'$ $\qquad \vec v = 3\vec i + \vec j$



$\vec v_x' = v_x \cos 60^\circ - v_y \sin 60^\circ = 3\frac{\sqrt3}{2} - \frac{1}{2}$

$\vec v_y' = v_x \sin 60^\circ + v_y \cos 60^\circ = \frac{3}{2} + \frac{\sqrt3}{2}$